TAME COHOMOLOGY AND ITS APPLICATIONS (LECTURES AT INSTITUTE OF SCIENCE TOKYO IN NOVEMBER, 2025)

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Contents

1. Review on topos theory

1.1. **Functoriality of presheaves.** A functor $u : \mathcal{C} \to \mathcal{D}$ induces

$$u^p : \mathbf{PSh}(\mathcal{D}) \to \mathbf{PSh}(\mathcal{C})$$

given by $u^p F = F \circ u$, in other words $u^p F(V) = F(u(V))$ for $V \in \mathcal{C}$.

Proposition 1.1. *There exists a functor called the left Kan extension of F along u*

$$u_p : \mathbf{PSh}(\mathcal{C}) \to \mathbf{PSh}(\mathcal{D})$$

which is a left adjoint to the functor u^p. In other words

$$\mathrm{Hom}_{\mathbf{PSh}(\mathcal{C})}(F, u^p G) = \mathrm{Hom}_{\mathbf{PSh}(\mathcal{D})}(u_p F, G)$$

holds bifunctorially in $F \in \mathbf{PSh}(\mathcal{C})$ and $G \in \mathbf{PSh}(\mathcal{D})$.

For $V \in \mathcal{D}$, let $I^u(V)$ denote the category whose objects are pairs (U, φ) with $U \in \mathcal{C}$ and $\varphi : V \to u(U)$ and

$$\mathrm{Hom}_{I^u(V)}((U,\varphi),(U',\varphi')) = \{f : U \to U' \text{ in } \mathcal{C} |\ u(f) \circ \varphi = \varphi'\}.$$

We sometimes drop the superscript u from the notation and we simply write $I(V)$. For $F \in \mathbf{PSh}(\mathcal{C})$, we define

$$u_p F(V) = \varinjlim_{(U,\varphi)\in I(V)^{op}} F(U) = \varinjlim_{I(V)^{op}} F_V,$$

where $F_V \in \mathbf{PSh}(I(V), \mathbf{Sets})$ given by

$$F_V : I(V)^{op} \to \mathbf{Sets}\ :\ (U,\varphi) \to F(U).$$

To show that $u_p F \in \mathbf{PSh}(\mathcal{D})$, note that for $g : V' \to V$ in $\mathcal{D}$, we get a functor $g : I(V) \to I(V')$ by setting $g(U,\varphi) = (U, \varphi \circ g)$. It induces a map

$$u_p F(V) = \varinjlim_{(U,\varphi)\in I(V)^{op}} F(U) \to \varinjlim_{(W,\psi)\in I(V')^{op}} F(W) = u_p F(V').$$

A map of $F \to F'$ in $\mathbf{PSh}(\mathcal{C})$ induces for $V \in \mathcal{D}$

$$u_pF(V) = \varinjlim_{(U,\varphi)\in I(V)^{op}} F(U) \to \varinjlim_{(U,\varphi)\in I(V)^{op}} F'(U) = u_pF(V).$$

Thus, we have defined a functor

$$u_p : \mathbf{PSh}(\mathcal{C}) \to \mathbf{PSh}(\mathcal{D}).$$

To show that

$$\mathrm{Hom}_{\mathbf{PSh}(\mathcal{C})}(F, u^pG) = \mathrm{Hom}_{\mathbf{PSh}(\mathcal{D})}(u_pF, G)$$

holds bifunctorially in F and G.

Lemma 1.2. *Let $u : \mathcal{C} \to \mathcal{D}$ be a functor. Assume*

(i) $\mathcal{C}$ has a final object e and $u(e)$ is a final object of $\mathcal{D}$,
(ii) $\mathcal{C}$ admits fiber products and u commutes with them.

Then, u_p commutes with fintie limits.

Proof. This follows from the fact that the categories $I^u(V)^{op}$ are filtered by [28, 00X3]. □

1.2. **Sites and sheaves.** .

Definition 1.3. A site is given by a pair $(\mathcal{C}, \tau)$ of a category $\mathcal{C}$ and a Grothendieck pretopology τ which is a function assigning to each object $U \in \mathcal{C}$ a collection $\mathrm{Cov}(U)$ of families of morphisms $\{U_i \to U\}_{i\in I}$, called coverings family of U, satisfying the following axioms:

- (i) If $V \to U$ is an isomorphism, we have $\{V \to U\} \in \mathrm{Cov}(U)$.
- (ii) If $\{U_i \to U\}_{i\in I} \in \mathrm{Cov}(U)$ and $\{V_{ij} \to U_i\}_{jinJ_i} \in \mathrm{Cov}(U_i)$ for each $i \in I$, we have then $\{V_{ij} \to U\}_{i\in I, j\in J_i} \in \mathrm{Cov}(U)$.
- (iii) If $\{U_i \to U\}_{i\in I} \in \mathrm{Cov}(U)$ and $V \to U$ is a morphism of $\mathcal{C}$, then $U_i \times_U V$ exists for all $i \in I$ and we have $\{U_i \times_U V \to V\}_{i\in I} \in \mathrm{Cov}(V)$.

Example 1.4. For a scheme S, let $\mathbf{Sch}_S$ be the category of schemes of finite presentation over S.

- (i) Let $\mathrm{\acute{E}t}_S$ be the full subcategory of $\mathbf{Sch}_S$ of étale schemes over S. The big étale site $(\mathbf{Sch}_S)_{\text{ét}}$ is the site whose underlying category is $\mathbf{Sch}_S$ and whose coverings are étale covering[1]. The small étale site $(\mathbf{Sch}_X)_{\text{ét}}$ is the full subcategory of $(\mathbf{Sch}_S)_{\text{ét}}$ whose objects are those U/S such that $U \to S$ is étale. A covering of $S_{\text{ét}}$ is any étale covering $\{U_i \to U\}$ with $U \in S_{\text{ét}}$.

Definition 1.5. Let $\mathcal{C}$ be a site, and let F be a presheaf of sets on $\mathcal{C}$. We say F is a sheaf if for every $U \in \mathcal{C}$ and every covering $\{U_i \to U\}_{i\in I} \in \mathrm{Cov}(U)$ the diagram

$$F(U) \to \prod_{i\in I} F(U_i) \underset{pr_1^*}{\overset{pr_0^*}{\rightrightarrows}} \prod_{(i_0,i_1)\in I\times I} F(U_{i_0} \times_U U_{i_1})$$

represents the first arrow as the equalizer of pr_0^* and pr_1^*. We let $\mathbf{Shv}(\mathcal{C}) \subset \mathbf{PSh}(\mathcal{C})$ denote the full subcategory of sheaves (of sets).

Lemma 1.6. *Let $\mathcal{F} : I \to \mathbf{Shv}(\mathcal{C})$ be a diagram. Then $\varprojlim_I \mathcal{F}$ exists and is equal to the limit in $\mathbf{PSh}(\mathcal{C})$.*

Proposition 1.7. *There exists a functor called the sheafification*

$$a : \mathbf{PSh}(\mathcal{C}) \to \mathbf{Shv}(\mathcal{C})$$

which is a left adjoint to the inclusion functor $\mathbf{PSh}(\mathcal{C}) \to \mathbf{Shv}(\mathcal{C})$. In other words

$$\mathrm{Hom}_{\mathbf{PSh}(\mathcal{C})}(F, G) = \mathrm{Hom}_{\mathbf{Shv}(\mathcal{C})}(aF, G)$$

[1]For $T \in \mathbf{Sch}_S$, an étale covering of T is a family of morphisms $\{f_i : T_i \to T\}_{i\in I}$ in $\mathbf{Sch}_S$ such that each f_i is étale and $T = \cup f_i(T_i)$.

holds bifunctorially in $F \in \mathbf{PSh}(\mathcal{C})$ and $G \in \mathbf{Shv}(\mathcal{C})$. Moreover, a is exact.

Let $F \in \mathbf{PSh}(\mathcal{C})$. For $\mathfrak{U} = \{U_i \to U\}_{i \in I} \in \mathrm{Cov}(U)$, put

$$H^0(\mathfrak{U}, F) = \mathrm{equalizer}\Big(\prod_{i \in I} F(U_i) \underset{pr_1^*}{\overset{pr_0^*}{\rightrightarrows}} \prod_{(i_0, i_1) \in I \times I} F(U_{i_0} \times_U U_{i_1})\Big)$$

There is a canonical map $F(U) \to H^0(\mathfrak{U}, F)$[2].

For $U \in \mathcal{C}$, let $\mathrm{Cov}(U)$ be the category of all coverings of U in $\mathcal{C}$ whose morphisms are the refinements (see §1.5). Note that $\mathrm{Cov}(U)$ is not empty since $\{id : U \to U\}$ is an object of it. By definition the construction $\mathfrak{U} \mapsto H^0(\mathfrak{U}, F)$ is an object of $\mathbf{PSh}(\mathrm{Cov}(U))$. For $F \in \mathbf{PSh}(\mathcal{C})$, we define

$$F^+(U) = \varinjlim_{\mathfrak{U} \in \mathrm{Cov}(U)^{op}} H^0(\mathfrak{U}, F).$$

Note that $F^+(U)$ is the zeroth Čech cohomology of F over U (see §1.5).

Lemma 1.8. *(1) For $F \in \mathbf{PSh}(\mathcal{C})$, F^+ is an object of $\mathbf{PSh}(\mathcal{C})$ equipped with a canonical map $F \to F^+$ in $\mathbf{PSh}(\mathcal{C})$. Moreover, the construction is functorial, i.e. a map $f : F \to G$ in $\mathbf{PSh}(\mathcal{C})$ induces a map $f^+ : F^+ \to G^+$ such that the following diagram commutes in $\mathbf{PSh}(\mathcal{C})$:*

$$\begin{array}{ccc} F & \longrightarrow & F^+ \\ \downarrow{\scriptstyle f} & & \downarrow{\scriptstyle f^+} \\ G & \longrightarrow & G^+ \end{array}$$

(2) The presheaf F^+ is separated.

Proof. [28, 00WB]. □

Proposition 1.9. *For $F \in \mathbf{PSh}(\mathcal{C})$, $(F^+)^+ \in \mathbf{Shv}(\mathcal{C})$ and the induced functor*

$$a = ((-)^+)^+ : \mathbf{PSh}(\mathcal{C}) \to \mathbf{Shv}(\mathcal{C})$$

is a left adjoint to the inclusion functor $\mathbf{PSh}(\mathcal{C}) \to \mathbf{Shv}(\mathcal{C})$. Moreover, a is exact.

Proof. [28, 00WB]. The exactness of a follows from the fact that $\mathrm{Cov}(U)$ is filtered (the point is to show a commutes with finite limits). □

1.3. **Functoriality of sheaves.**

Definition 1.10. Let $\mathcal{C}$ and $\mathcal{D}$ be sites. A functor $u : \mathcal{C} \to \mathcal{D}$ is called continuous if for every $V \in \mathcal{C}$ and every $\{V_i \to V\}_{i \in I} \in \mathrm{Cov}(V)$, we have the following

(i) $\{u(V_i) \to u(V)\}_{i \in I} \in \mathrm{Cov}(u(V))$,
(ii) for any morphism $T \to V$ in $\mathcal{C}$, the morphism $u(T \times_V V_i) \to u(T) \times_{u(V)} u(V_i)$ is an isomorphism.

Example 1.11. For a map $f : T \to S$ of schemes, consider

$$u : \mathrm{\acute{E}t}_S \to \mathrm{\acute{E}t}_T \;:\; X \to X \times_S T.$$

Then, u is continuous for the étale topology.

Lemma 1.12. *If $u : \mathcal{C} \to \mathcal{D}$ is continuous, u^p induces*

$$u^s : \mathbf{Shv}(\mathcal{D}) \to \mathbf{Shv}(\mathcal{C}).$$

Proof. Exercise. □

[2] This is the zeroth Čech cohomology of F over U with respect to the covering $\mathfrak{U}$.

Lemma 1.13. *If $u : \mathcal{C} \to \mathcal{D}$ is continuous, the functor*

$$u_s : \mathbf{Shv}(\mathcal{D}) \to \mathbf{Shv}(\mathcal{C}) \ : \ G \to a(u_p(G))$$

is a left adjoint to u^s.

Proof. Follows directly from Propositions 1.9 and 1.1. □

Definition 1.14. Let $\mathcal{C}$ and $\mathcal{D}$be sites. A morphism of sites $f : \mathcal{D} \to \mathcal{C}$ is given by a continuous functor $u : \mathcal{C} \to \mathcal{D}$ such that the functor u_s is exact.

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Proposition 1.15. *Let $u : \mathcal{C} \to \mathcal{D}$ be a continuous morphism of sites. Assume*

(i) $\mathcal{C}$ has a final object e and $u(e)$ is a final object of $\mathcal{D}$,
(ii) $\mathcal{C}$ admits fiber products and u commutes with them.

Then, u defines a morphism of sites, i.e. u_s is exact.

Proof. This follows from Lemma 1.2 and the exactness of a from Proposition 1.9 (see [28, 00X6]). □

Definition 1.16. A topos is the category $\mathbf{Shv}(\mathcal{C})$ of sheaves on a site $\mathcal{C}$.

(1) Let $\mathcal{C}$, $\mathcal{D}$ be sites. A morphism of topoi $f : \mathbf{Shv}(\mathcal{D}) \to \mathbf{Shv}(\mathcal{C})$ is given by a adjoint pair of functors

$$f^* : \mathbf{Shv}(\mathcal{C}) \leftrightarrows \mathbf{Shv}(\mathcal{D}) : f_*,$$

namely we have for $G \in \mathbf{Shv}(\mathcal{C})$ and $F \in \mathbf{Shv}(\mathcal{D})$

$$\mathrm{Hom}_{\mathbf{Shv}(\mathcal{D})}(f^*G, F) = \mathrm{Hom}_{\mathbf{Shv}(\mathcal{C})}(G, f_*F)$$

bifunctorially, and the functor f^* commutes with finite limits, i.e., is left exact.

(2) Let $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}$ be sites. Given morphisms of topoi $f : \mathbf{Shv}(\mathcal{D}) \to \mathbf{Shv}(\mathcal{C})$ and $g : \mathbf{Shv}(\mathcal{E}) \to \mathbf{Shv}(\mathcal{D})$, the composition $f \circ g$ is the morphism of topoi defined by the functors $(f \circ g)_* = f_* \circ g_*$ and $(f \circ g)^* = g^* \circ f^*$.

Lemma 1.17. *Given a morphism of sites $f : D \to C$ corresponding to the functor $u : \mathcal{C} \to \mathcal{D}$, the pair of functors $(f^* = u_s, f_* = u^s)$ is a morphism of topoi.*

Proof. This is obvious from Definition 1.14. □

1.4. **Cohomology.**

Theorem 1. *Let $\mathcal{C}$ be a site. Then, the category $\mathbf{Shv}(\mathcal{C}, \mathbf{Ab})$ of abelian sheaves on a site is an abelian category which has enough injectives.*

Proof. [28, 03NU]. □

By the theorem, we can define cohomology as the right-derived functors of the sections functor $F \to F(U)$ for $U \in \mathcal{C}$ and $F \in \mathbf{Shv}(\mathcal{C}, \mathbf{Ab})$ defined as

$$H^i(U, F) := R^i\Gamma(U, F) = H^i(\Gamma(U, I^\bullet)),$$

where $F \to I^\bullet$ is an injective resolution. To do this, we should check that the functor $\Gamma(U, -)$ is left exact. This is true and is part of why the category $\mathbf{Shv}(\mathcal{C}\mathbf{Ab})$ is abelian, see Modules on Sites, Lemma 3.1. For more general discussion of cohomology on sites (including the global sections functor and its right derived functors), see Cohomology on Sites, Section 2. The family of functors $H^i(U, -)$ forms a universal δ-functor $\mathbf{Shv}(\mathcal{C}, \mathbf{Ab}) \to \mathbf{Ab}$.

It sometimes happens that the site $\mathbf{C}$ does not have a final object. In this case, we define the global sections of $F \in \mathbf{PSh}(\mathcal{C}, S_{\text{ét}})$ over $\mathbf{C}$ to be the set

$$\Gamma(\mathcal{C}, F) = \mathrm{Hom}_{\mathbf{PSh}(\mathcal{C})}(e, F),$$

where e is a final object in $\mathbf{PSh}(\mathcal{C}, \mathbf{Sets})$. In this case, given $F \in \mathbf{Shv}(\mathcal{C}, \mathbf{Ab})$, we define the i-th cohomology group of Fon $\mathbf{C}$ as follows

$$H^i(\mathcal{C}, F) = H^i(\Gamma(\mathcal{C}, I^\bullet))).$$

In other words, it is the i-th right derived functor of the global sections functor. The family of functors $H^i(\mathcal{C}, -)$ forms a universal δ-functor $\mathbf{Shv}(\mathcal{C}, \mathbf{Ab}) \to \mathbf{Ab}$.

1.5. **Čech cohomology.** For $U \in \mathcal{C}$ and $\mathfrak{U} = \{U_i \to U\}_{i \in I} \in \mathrm{Cov}(U)$, write $U_{i_0 \dots i_p} = U_{i_0} \times_U \cdots \times_U U_{i_p}$ for the $(p+1)$-fold fiber product over U of members of $\mathfrak{U}$. Let $F \in \mathbf{PSh}(\mathcal{C}, \mathbf{Ab})$, set

$$\check{C}^p(\mathfrak{U}, F) = \prod_{(i_0 \dots i_p) \in I^{p+1}} F(U_{i_0 \dots i_p}).$$

For $s \in \check{C}^p(\mathfrak{U}, F)$, we denote $s_{i_0 \dots i_p}$ its value in $F(U_{i_0 \dots i_p})$. We define

$$d : \check{C}^p(\mathfrak{U}, F) \to \check{C}^{p+1}(\mathfrak{U}, F)$$

by the formula

$$d(s)_{i_0 \dots i_{p+1}} = \sum_{j=0}^{p+1} (-1)^j (s_{i_0 \dots \hat{i_j} \dots i_{p+1}})_{|U_{i_0 \cdots i_{p+1}}}.$$

It is straightforward to see that $d \circ d = 0$, i.e. $\check{C}(\mathfrak{U}, F)$ is a complex, which we call Čech complex associated to F and $\mathfrak{U}$. Its cohomology groups

$$\check{H}^i(\mathfrak{U}, F) = H^i(\check{C}(\mathfrak{U}, F))$$

are called the Čech cohomology groups associated to F and $\mathfrak{U}$.

Lemma 1.18. *For $U \in \mathcal{C}$ and $\mathfrak{U} = \{U_i \to U\}_{i \in I} \in \mathrm{Cov}(U)$, there is a transformation of functors:*

$$\mathbf{Shv}(\mathcal{C}, \mathbf{Ab}) \to \mathcal{D}(\mathbb{Z}) \ : \ \check{C}(\mathfrak{U}, -) \to R\Gamma(U, -).$$

Moreover, there is a spectral sequence for $F \in \mathbf{Shv}(\mathcal{C}, \mathbf{Ab})$:

$$E_2^{p,q} = \check{H}^p(\mathfrak{U}, \mathcal{H}^q(F)) \Rightarrow H^{p+q}(U, F),$$

which is functorial in F, where $\mathcal{H}^q(F) \in \mathbf{PSh}((X, \tilde{X})_t, \mathbf{Ab})$ is given by $\mathcal{U} \to H^q_t(\mathcal{U}, F)$. In particular, if $H^i(U_{i_0} \times_U \cdots \times_U U_{i_p}, F) = 0$ for all $i > 0$, $p \geq 0$ and $i_0, \dots, i_p \in I$, then we have $\check{H}^p(\mathfrak{U}, F) = H^p(U, F)$.

Proof. [28, 03AX, 03AZ, 03F7]. □

For coverings $\mathfrak{U} = \{U_i \to U\}_{i \in I}$ and $\mathfrak{V} = \{V_j \to V\}_{j \in J}$ in $\mathcal{C}$, a morphism $\mathfrak{U} \to \mathfrak{V}$ is given by a morphism $U \to V$ in $\mathcal{C}$, a map of sets $\alpha : I \to J$ and for each $i \in I$ a morphism $U_i \to V_{\alpha(i)}$ such that the diagram

$$\begin{array}{ccc} U_i & \longrightarrow & V_{\alpha(i)} \\ \downarrow & & \downarrow \\ U & \longrightarrow & V \end{array}$$

is commutative. In the special case $U = V$ and $U \to V$ is the identity, we call $\mathfrak{U}$ a refinement of $\mathfrak{V}$. A remark is that if the above $\mathfrak{V}$ is the empty family, i.e., if $J = \varnothing$, then no family $\mathfrak{U} = \{U_i \to V\}_{i \in I}$ with $I \neq \varnothing$ can refine $\mathfrak{V}$.

For $U \in \mathcal{C}$, let $\mathrm{Cov}(U)$ be the category of all coverings of U in $\mathcal{C}$ whose morphisms are the refinements[3]. Note that $\mathrm{Cov}(U)$ is not empty since $\{id : U \to U\}$ is an object of it. Take $F \in \mathbf{PSh}(\mathcal{C}, \mathbf{Ab})$. By definition the construction $\mathfrak{U} \mapsto \check{C}(\mathfrak{U}, F)$ is a preshesaf on $\mathrm{Cov}(U)$ with values in the category of complexes of abelian groups. We define

$$\check{C}(U, F) := \varinjlim_{\mathfrak{U} \in \mathrm{Cov}(U)^{op}} \check{C}(\mathfrak{U}, F),$$

$$\check{H}^i(U, F) := H^i(\check{C}(\mathfrak{U}, F)) = \varinjlim_{\mathfrak{U} \in \mathrm{Cov}(U)^{op}} \check{\mathrm{H}}^i(\mathfrak{U}, F),$$

where the last equality holds since $\mathrm{Cov}(U)$ if cofiltered. By Lemma 1.18, we have a transformation of functors:

$$\mathbf{Shv}(\mathcal{C}, \mathbf{Ab}) \to \mathcal{D}(\mathbb{Z}) \ : \ \check{C}(U, -) \to R\Gamma(U, -).$$

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[3]By our conventions on sites this is indeed a category, i.e., the collection of objects and morphisms forms a set.

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